Graph Partitioning Problems in Neuroscience

CanaDAM 19

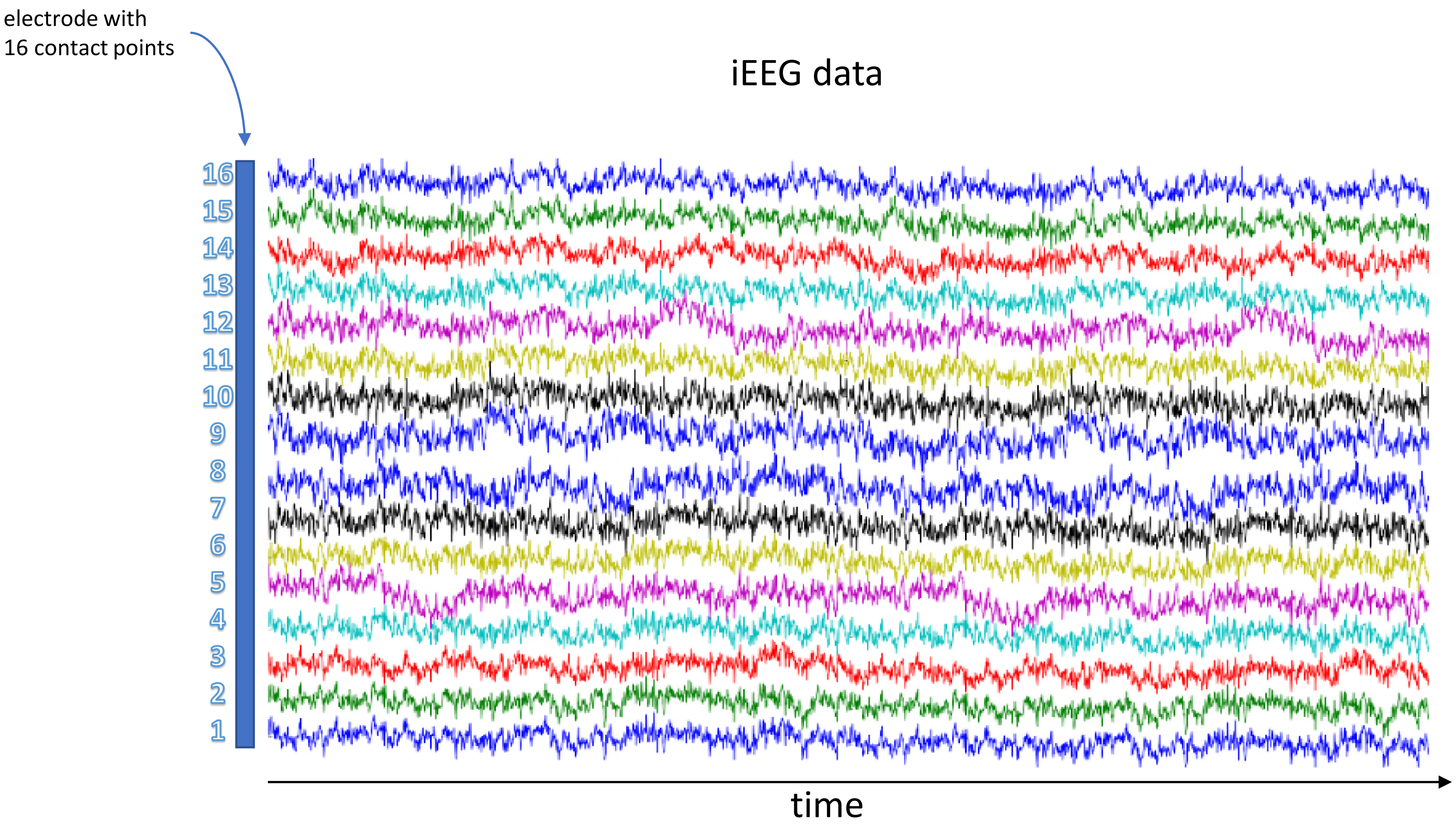
Joint work with:
Kris Vasudevan (University of Calgary)
Mike Cavers (University of Calgary)
Jordan Farrell (Stanford University)
Gordon Campbell Teskey (Hotchkiss Brain Institute, University of Calgary)

Keivan Hassani Monfared

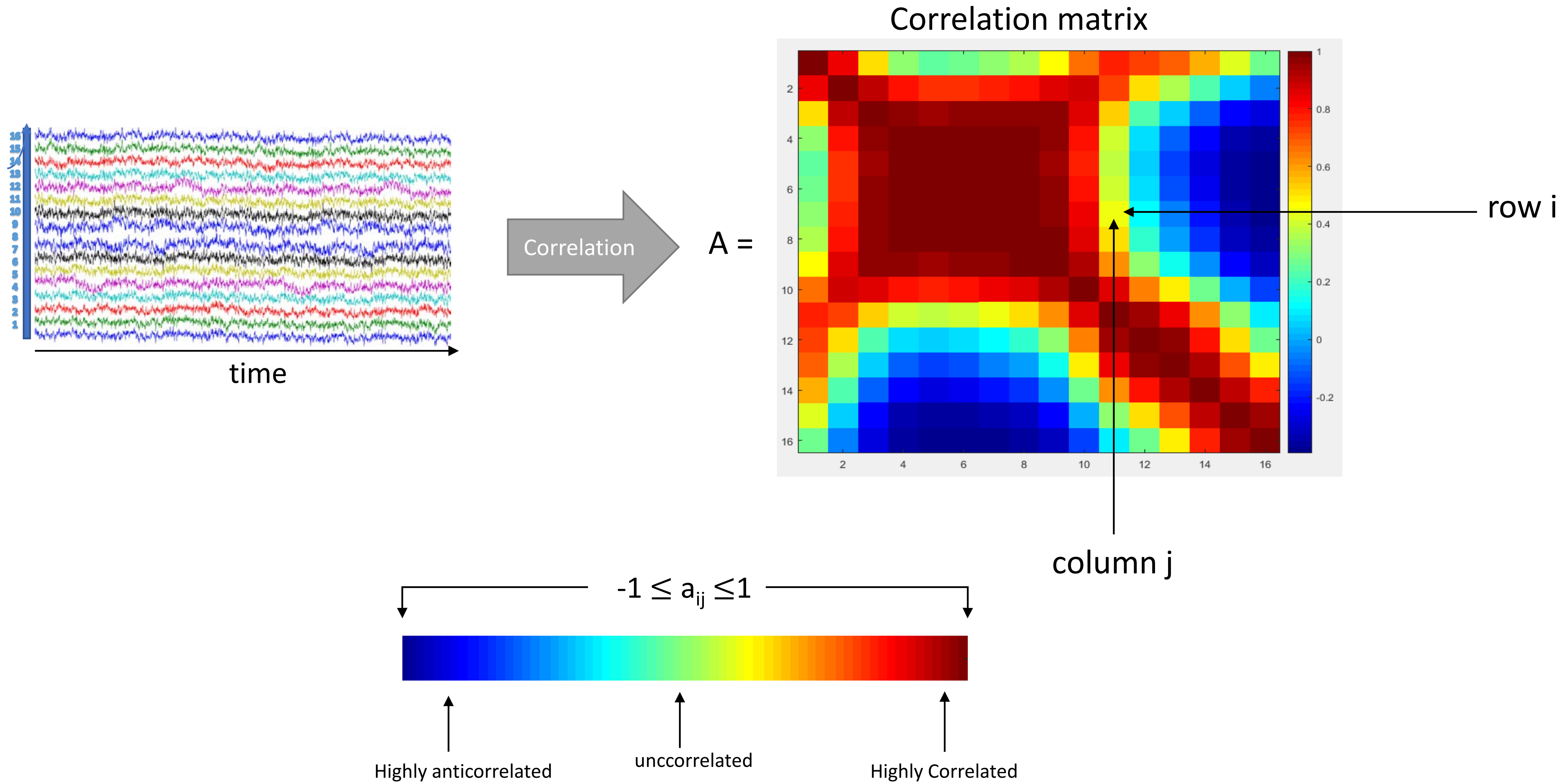
University of Victoria

k1monfared@gmail.com

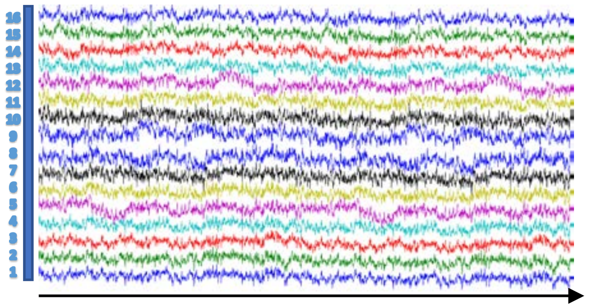
Data


electrode with
16 contact points
iEEG data
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
time

Model


time
Correlation
A =
Correlation matrix
row i
column j
-1 ≤ a_ij ≤1
Highly anticorrelated
unccorrelated
Highly Correlated

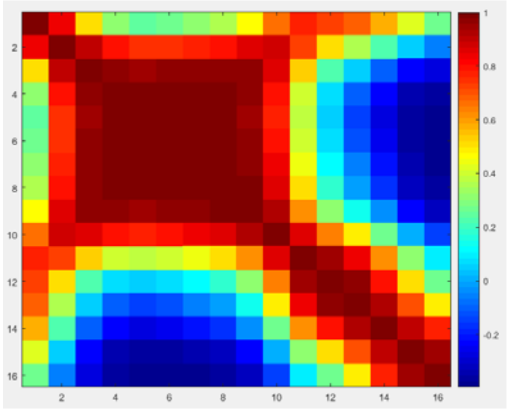
Model



time

Correlation

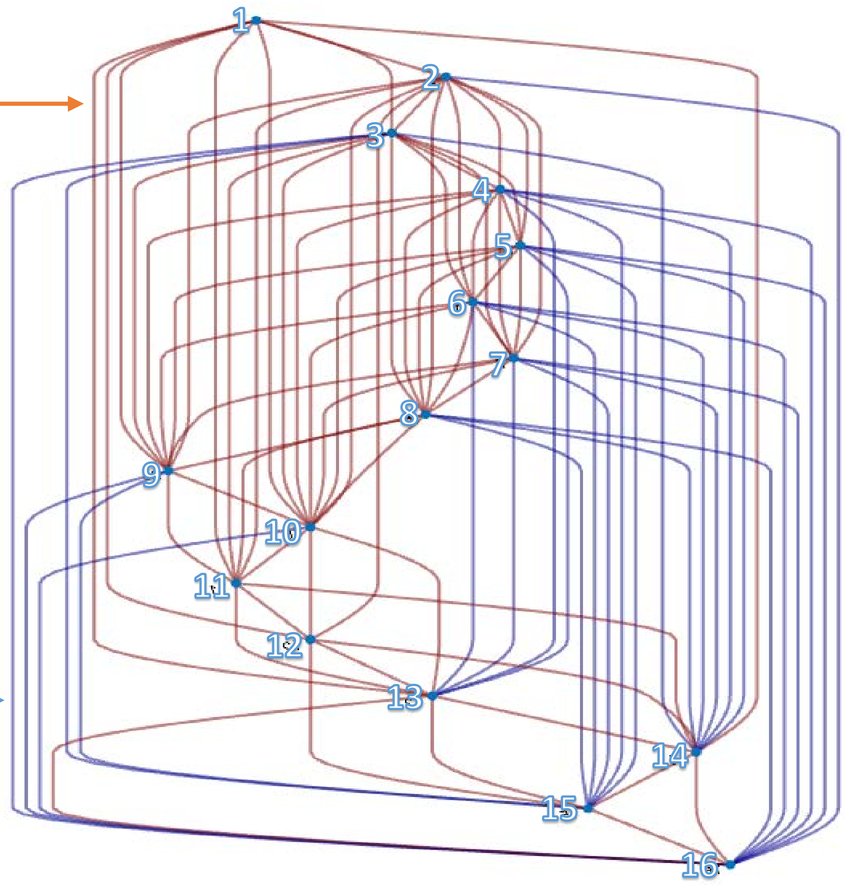
Correlation matrix



Graph

positive edges

negative edges

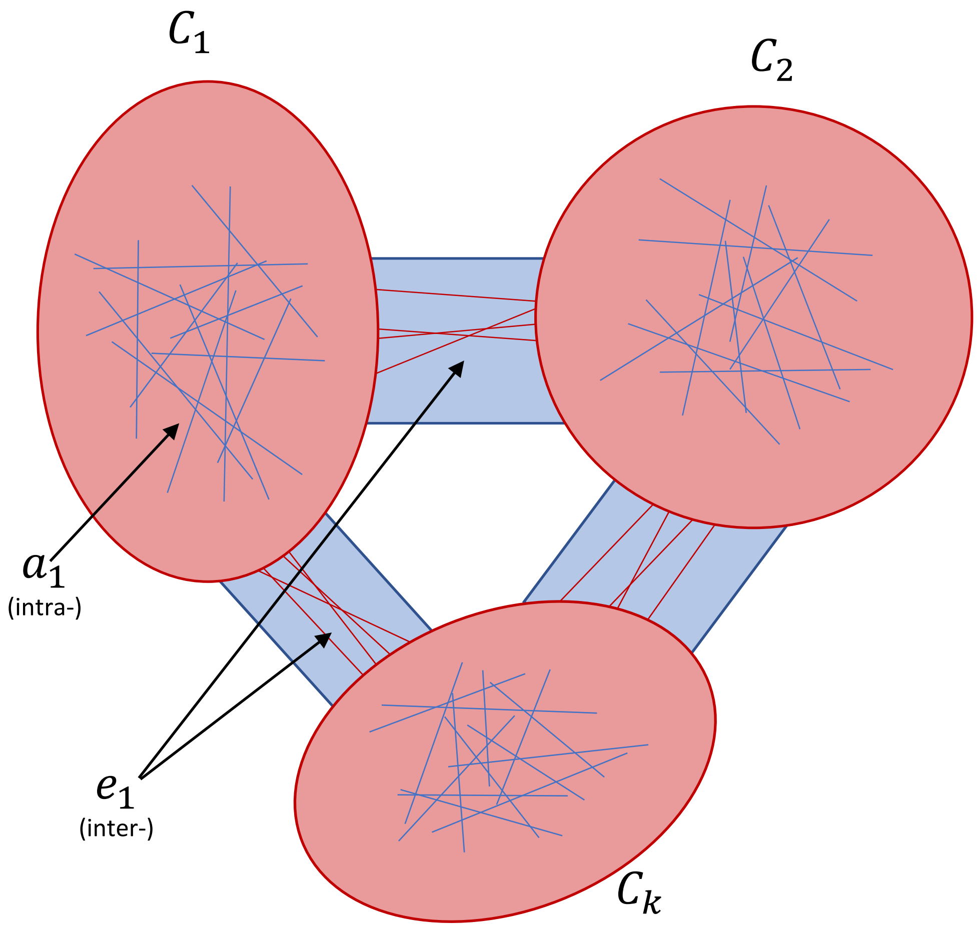


[Bullmore, Sporns, Nature Review, 2009]

Clustering problem (informal definition)

Find a partitioning $C = \{C_1, C_2, \ldots, C_k\}$ of the vertices of the graph so that:

1. Most of the positive edges are inside the partitions (intra-edges), and
2. Most of the negative edges are between the partitions (inter-edges).



Clustering problem (formal definition)

Girvan-Newman modularity for a given clustering $C_1, C_2, \ldots, C_k$ is defined by:

$$Q = \sum_{i=1}^{k} a_i - e_i^2$$

a_i: ratio of edges inside cluster i

e_i^2: ratio of edges between cluster i and other clusters

For **signed** graphs:

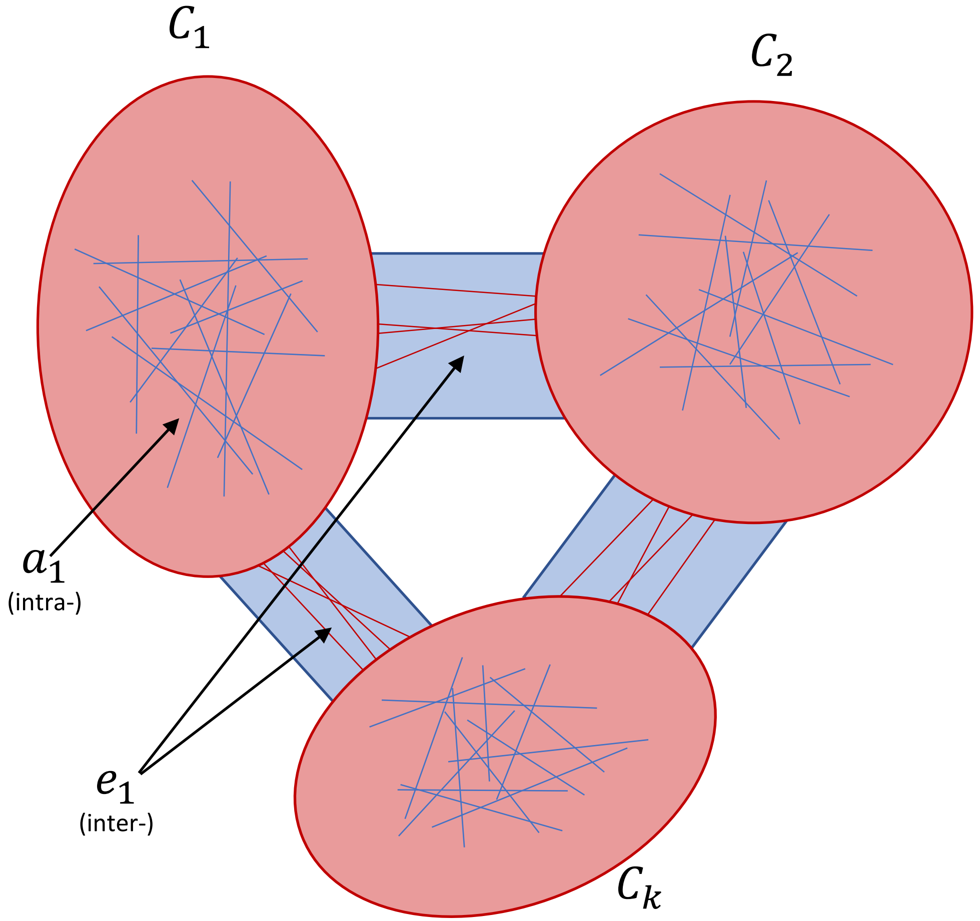
$$Q_s = \frac{m^+ Q^+ - m^- Q^-}{m^+ + m^-}$$

Q^+: modularity of positive edges

Q^-: modularity of negative edges

m^+: number of positive edges

m^-: number of negative edges



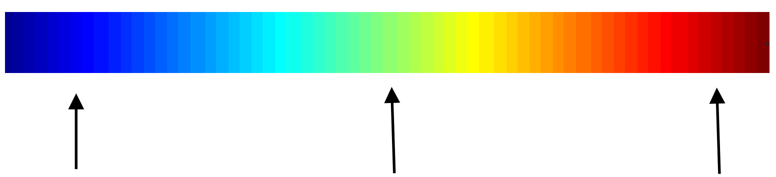
[Girvan, Newman, Physical Review, 2004]

Clustering problem (formal definition)

Find a partitioning $C = \{C_1, C_2, \dots, C_k\}$ of the vertices of the graph so that Q_s is maximized.

$$Q_s = \frac{m^+Q^+ - m^-Q^-}{m^+ + m^-}$$

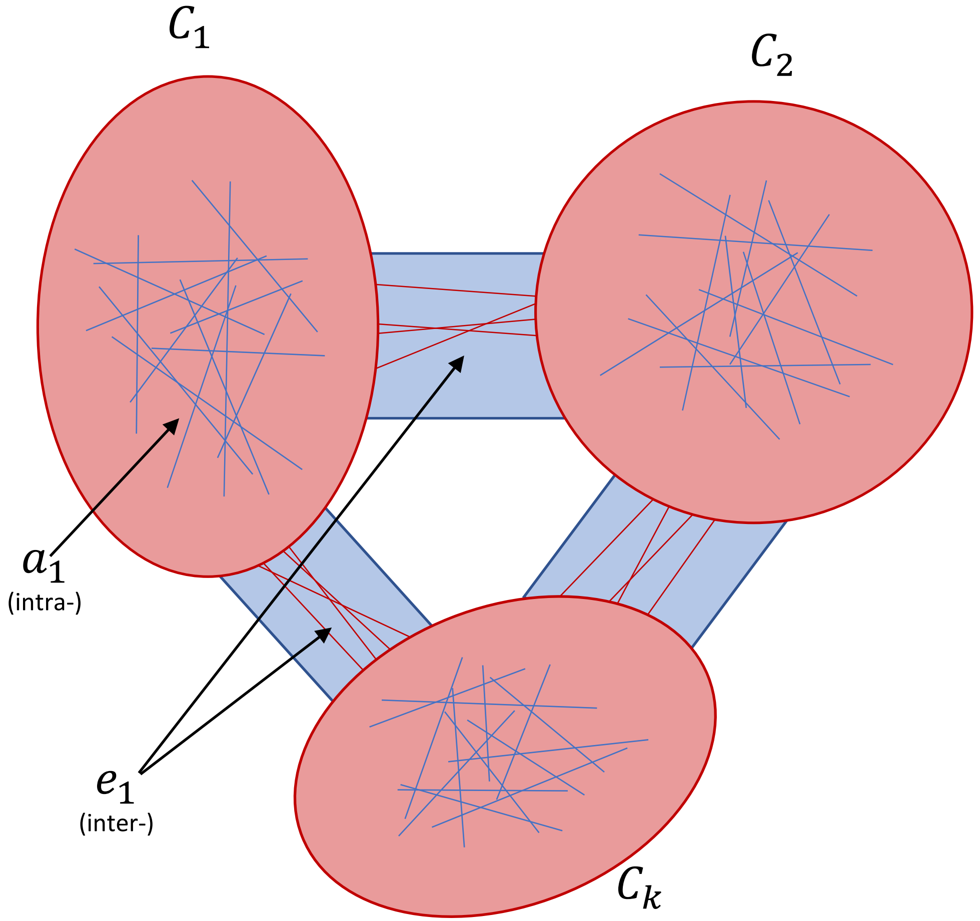
$$-2 \leq Q_s \leq 2$$



- no real communities
- highly anti-correlated intra-community interactions
- highly correlated inter-community interactions

As good as random or no clustering

- clear communities
- highly **correlated** **intra**-community interactions
- highly **anti-correlated** **inter**-community interactions

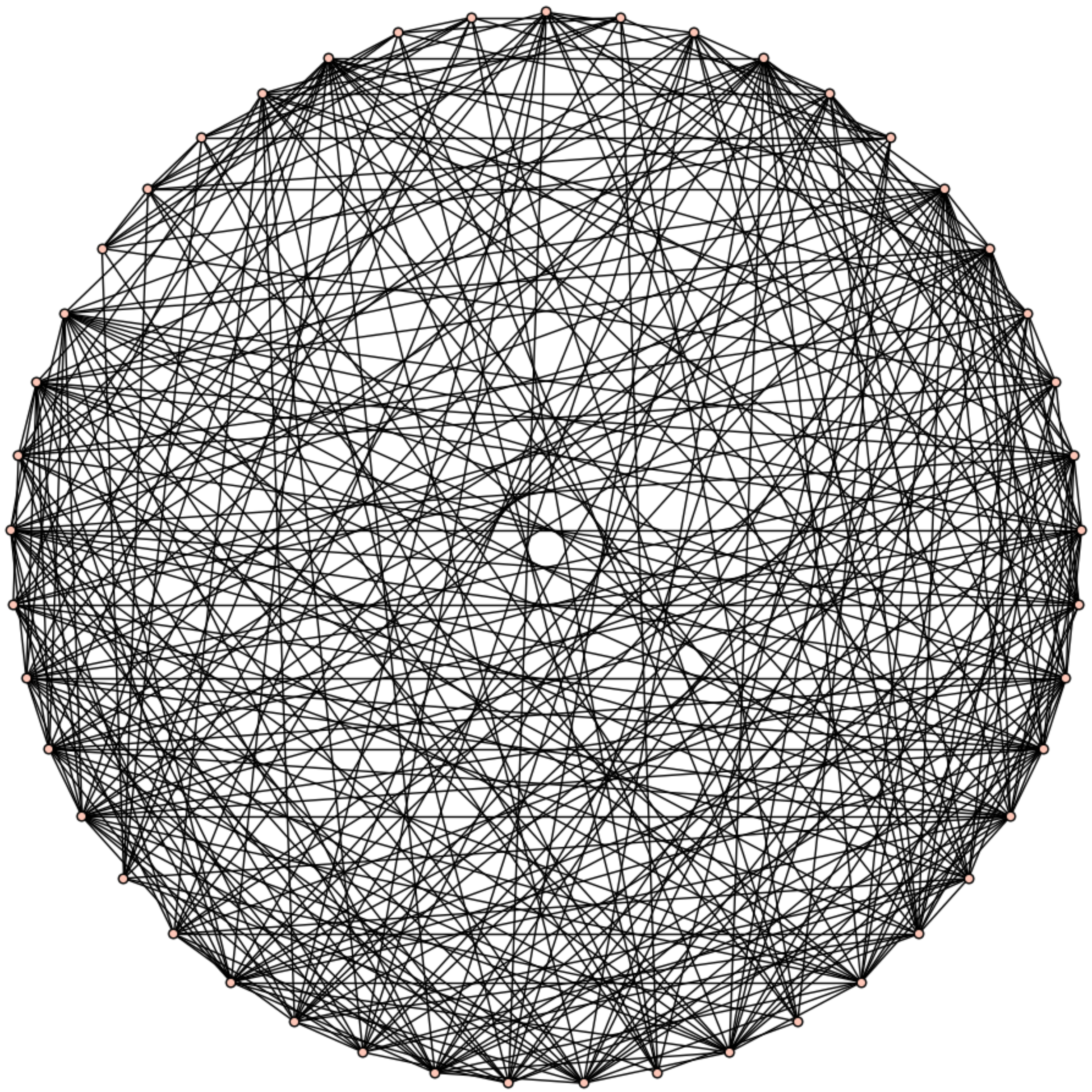


Method 1: Hierarchical Fiedler

1. Compute an eigenvector of the second smallest eigenvalue of the Laplacian matrix of the graph (the Fiedler vector)

2. Partition the vertices into two sets according to the sign of the corresponding entry in the Fiedler eigenvector

3. Repeat steps 1 and 2 for the partition with the smallest second eigenvalue of the Laplacian (smallest algebraic connectivity)



[Fiedler, Czech. Math. J. 1975]

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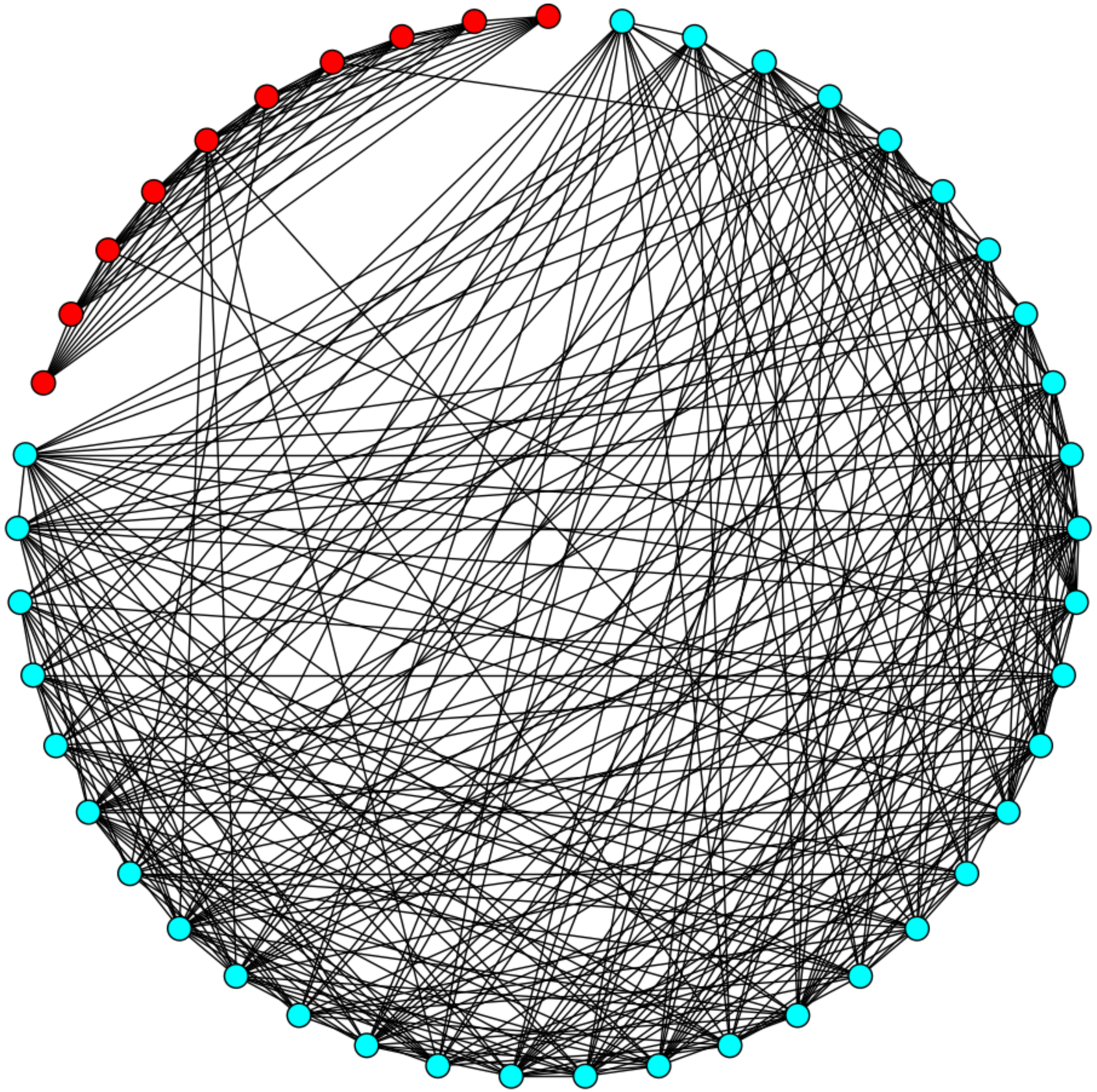


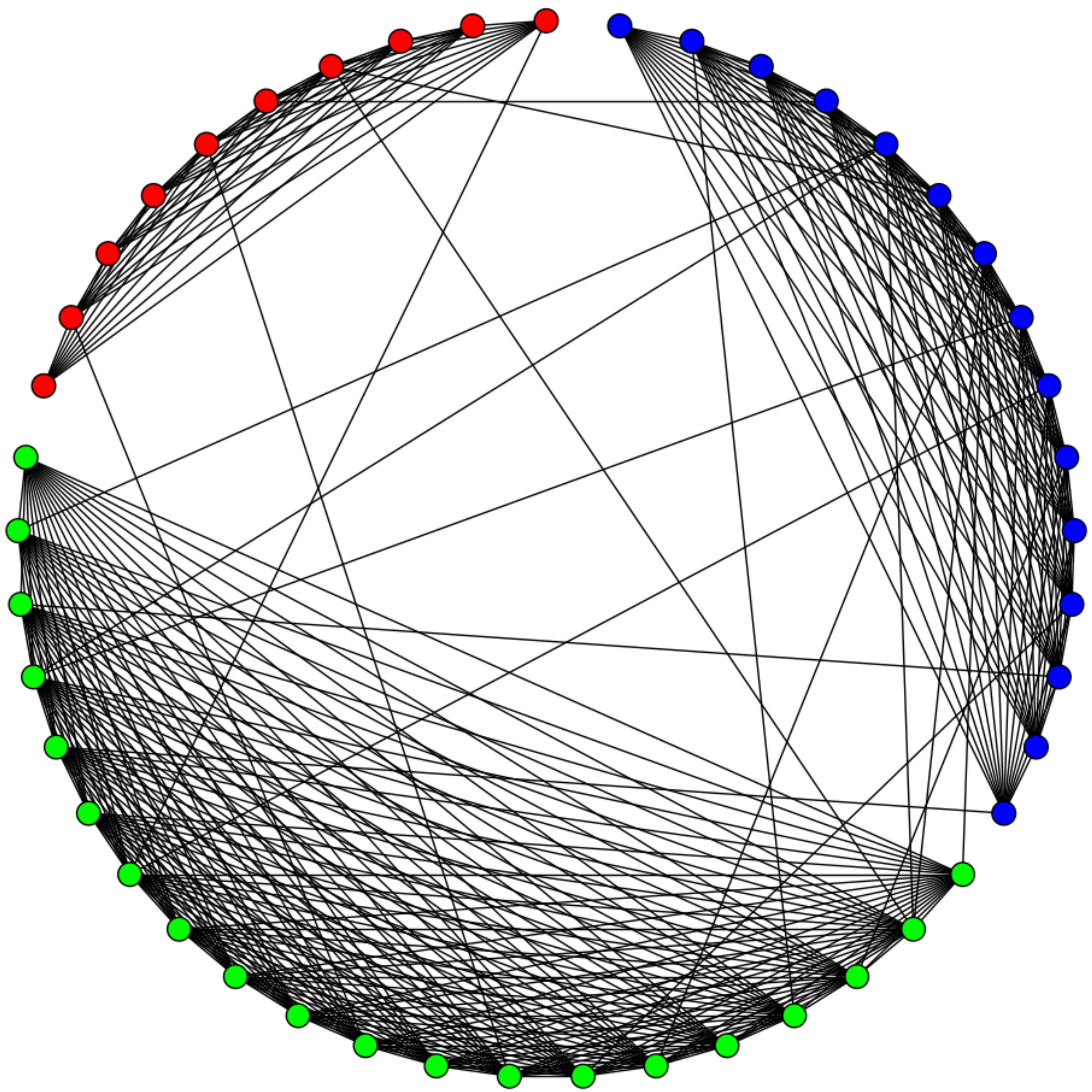
[Fiedler, Czech. Math. J. 1975]

Method 1: Hierarchical Fiedler (outline)

1. Compute an eigenvector of the second smallest eigenvalue of the Laplacian matrix of the graph (the Fiedler vector)

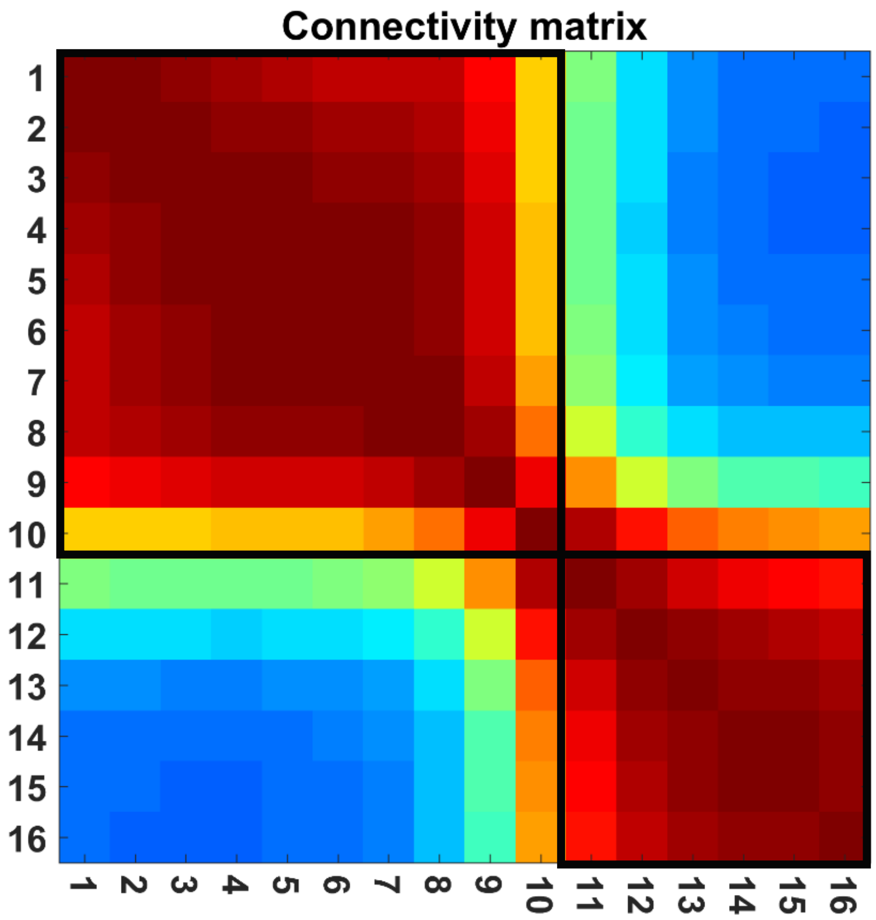
2. Partition the vertices into two sets according to the sign of the corresponding entry in the Fiedler eigenvector

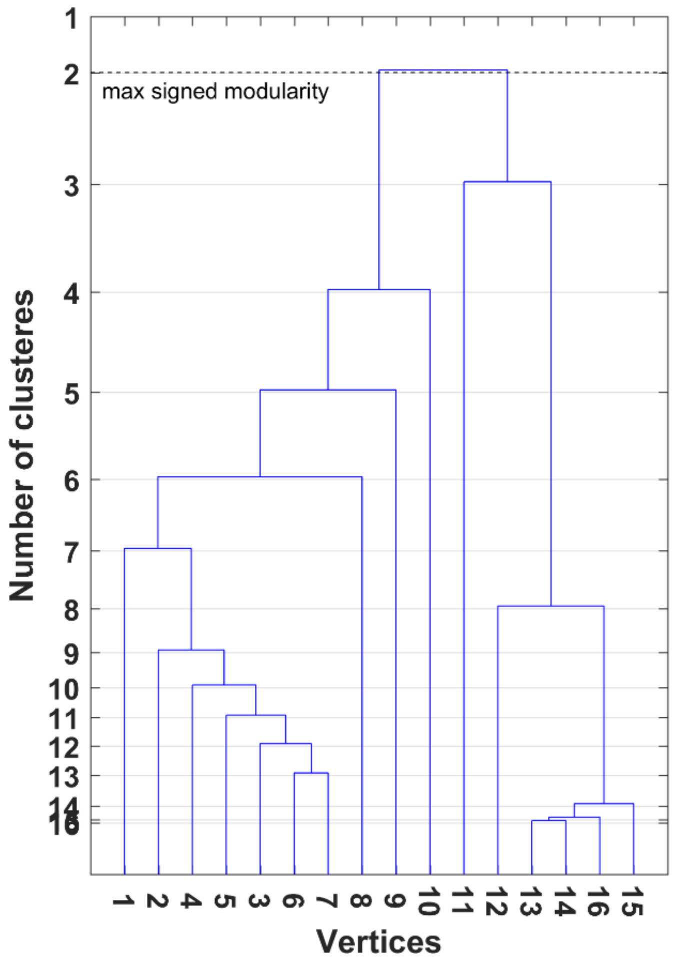
3. Repeat steps 1 and 2 for the partition with the smallest second eigenvalue of the Laplacian (smallest algebraic connectivity)

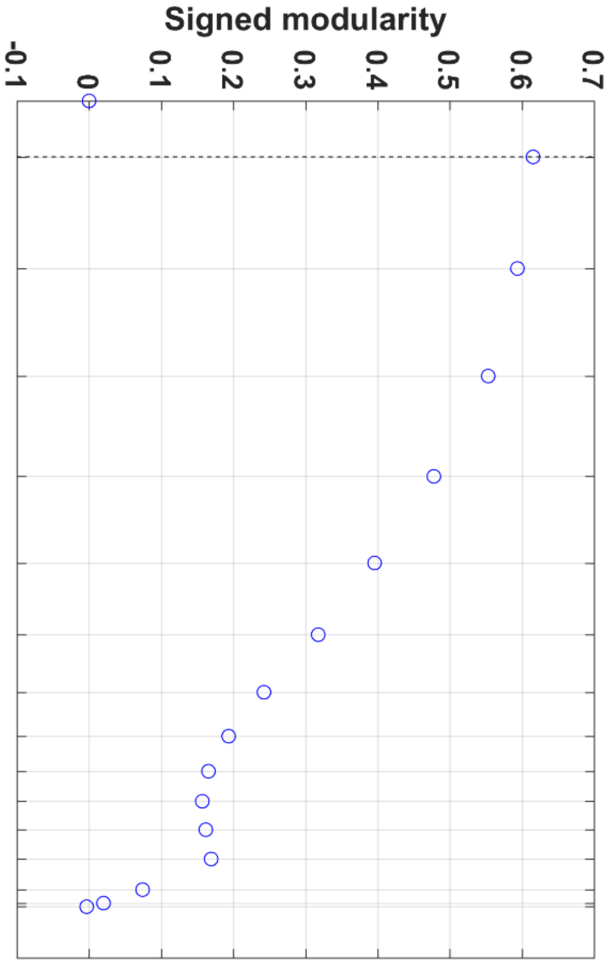


[Fiedler, Czech. Math. J. 1975]

Method 1: Hierarchical Fiedler (When to stop)

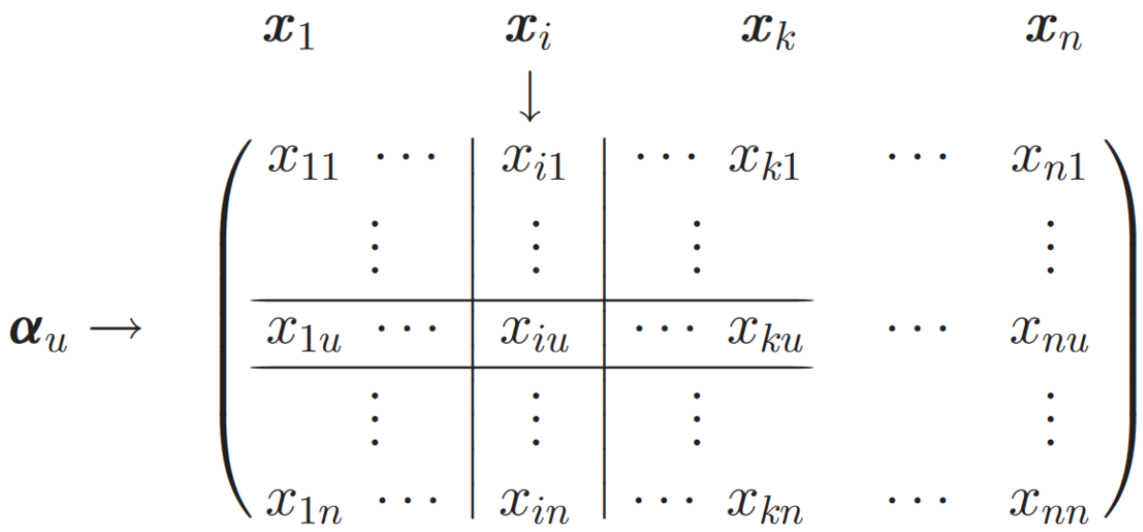






Method 2: Spectral Coordinates

$\boldsymbol{x_i}$: eigenvector corresponding to i-th largest eigenvalue of the adjacency matrix of the graph
$\boldsymbol{k}$: number of desired clusters
$\boldsymbol{\alpha_u}$: spectral coordinates of node u



[Wu, Wu, Lu, Li, IEEE Trans Knowledge Data Eng, 2017]

Method 2: Spectral Coordinates

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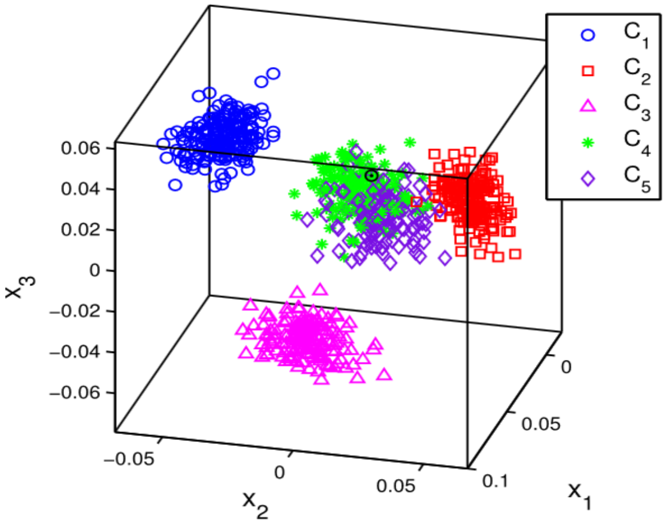
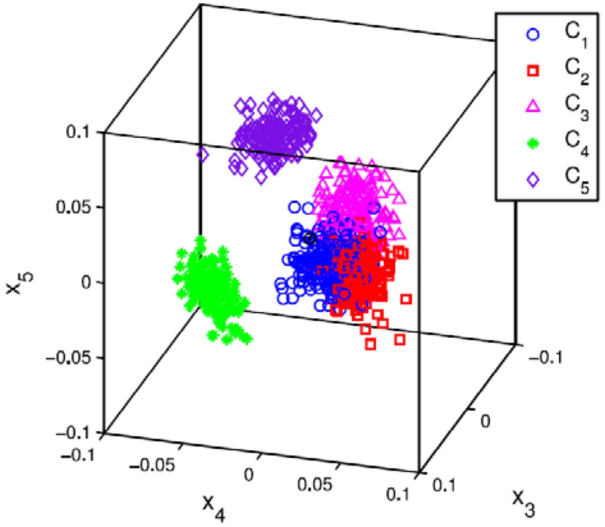
$$\begin{array}{ccccccc} \boldsymbol{x}_1 & & \boldsymbol{x}_i & & \boldsymbol{x}_k & & \boldsymbol{x}_n \\ & & \downarrow & & & & \end{array}$$

$$\boldsymbol{\alpha}_u \rightarrow \left(\begin{array}{cc|c|cccc} x_{11} & \cdots & x_{i1} & \cdots & x_{k1} & \cdots & x_{n1} \\ & \vdots & \vdots & \vdots & & & \vdots \\ \hline x_{1u} & \cdots & x_{iu} & \cdots & x_{ku} & \cdots & x_{nu} \\ \hline & \vdots & \vdots & \vdots & & & \vdots \\ x_{1n} & \cdots & x_{in} & \cdots & x_{kn} & \cdots & x_{nn} \end{array}\right)$$

- spectral coordinates of the nodes in the same communities tend to cluster together.
- Use a clustering algorithm, such as k-means to identify the nodes in the same clusters with various k.
- Choose the clustering with maximum signed modularity.





[Wu, Wu, Lu, Li, IEEE Trans Knowledge Data Eng, 2017]

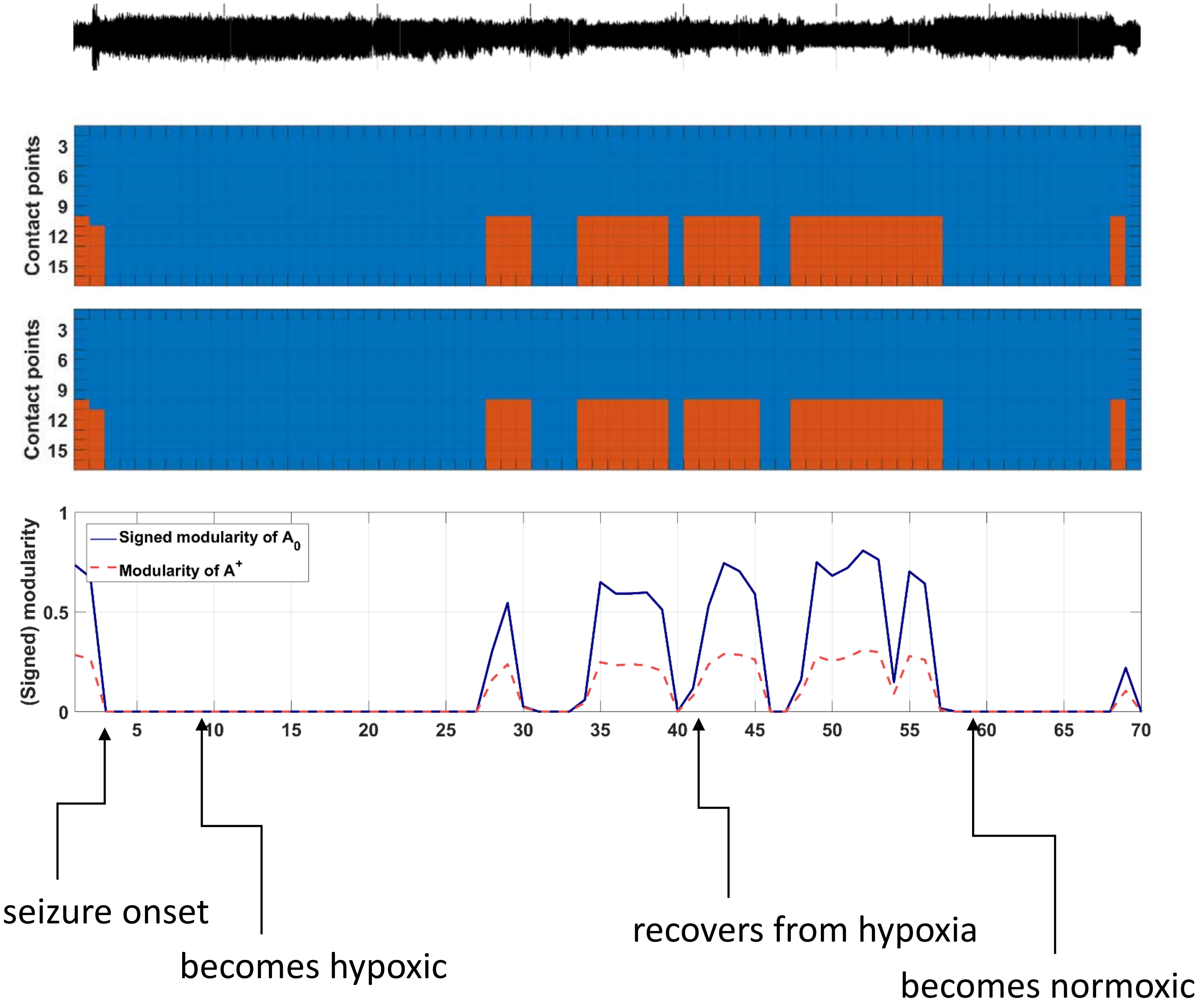
Results

One signal:

Clusters from Hierarchical Fiedler method:

Clusters from Spectral Coordinates method:

Signed modularity of both clusterings:



Community structure detection and evaluation during the pre- and post-ictal hippocampal depth recordings

[arxiv.org/abs/1804.01568]

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Thank you!

Keivan Hassani Monfared
University of Victoria
`k1monfared@gmail.com`

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