Stat 171 - Exam 3
SHOW ALL OF YOUR WORK

Name: Solution

1. You toss a coin until it lands head. What type of random variables are the followings:
   (a) Number of the times you tossed the coin. Discrete random variable
   (b) Amount of time you spent tossing the coin. Continuous random variable
2. The table below shows the number of tickets a police officer writes out each shift, and their probabilities.

x	0	1	2	3
$P(x)$	0.1	0.3	0.5	0.1

   (a) Verify that the table above represents a probability distribution.

For all $x: 0 \le P(x) \le 1$
and $\sum P(x) = .1 + .3 + .5 + .1 = 1$

   (b) Find the mean, variance, and the standard deviation.

$\mu = \sum x P(x) = 0 + .3 + 1 + .3 = 1.6$

x	$P(x)$	$x-\mu$	$(x-\mu)^2$	$(x-\mu)^2 \cdot P(x)$
0	.1	-1.6	2.56	0.256
1	.3	-0.6	0.36	0.108
2	.5	.4	0.16	0.080
3	.1	1.4	1.96	0.196
Σ	1			$0.640 = \sigma^2$

$\sigma = \sqrt{0.64} = 0.8$

3. About 20% of U.S. drivers are uninsured. You randomly select eight U.S. drivers and ask them whether they are uninsured. Find the probability that 5 of them are uninsured.

Binomial

$P(X=5) = {}_8C_5 (0.2)^5 (0.8)^3$

$= \frac{8!}{5!3!} \cdot (0.2)^5 (0.8)^3$

$= (56) \cdot (0.00032) \cdot (0.512) \approx 0.009$

4. The annual salary for taxi drivers is normally distributed with a mean about $30,000, and a standard deviation of $9000. A random taxi driver is selected. What is the probability that his/her salary is more than $20,000?

$P(x > 20000) = 1 - P(x < 20000)$

$= 1 - P(z < -1.11)$

$= 1 - 0.1335$

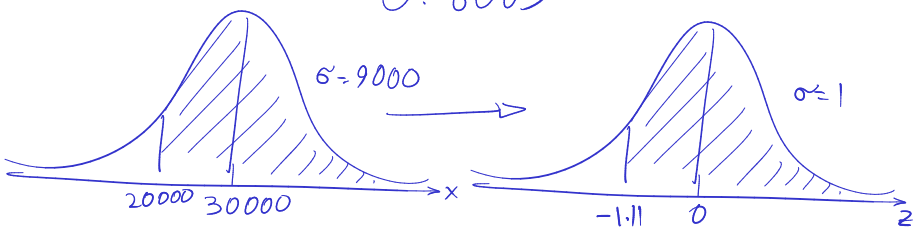
$= 0.8665$

$z = \frac{20000 - 30000}{9000} \approx -1.11$

$P(z < -1.11) = 0.1335$

Standard normal probabilities

z	.00	.01	.02
−3.4	.0003	.0003	.0003
−1.2	.1151	.1131	.1112
−1.1	.1357	.1335	.1314
−1.0	.1587	.1562	.1539



5. The annual salary for taxi drivers is normally distributed with a mean about $30,000, and a standard deviation of $9000. Nine random taxi drivers are selected. What is the probability that the average of their annual salary is more than $20,000?

$\mu_{\bar{x}} = 30000$

$\sigma_{\bar{x}} = \frac{9000}{\sqrt{9}} = 3000$

$\bar{z} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{20000 - 30000}{3000} \approx -3.33$

$P(\bar{z} < -3.33) =$

$P(\bar{x} > 20000) = 1 - P(\bar{x} < 20000)$

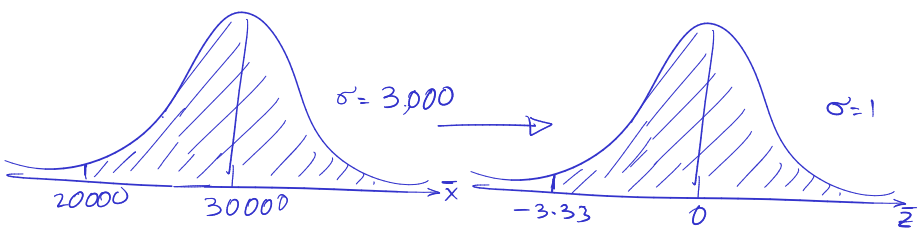
$= 1 - P(\bar{z} < -3.33)$

$= 1 - 0.0004$

$= 0.9996$

Standard normal probabilities

z	.00	.01	.02	.03	.04
−3.4	.0003	.0003	.0003	.0003	.0003
−3.3	.0005	.0005	.0005	.0004	.0004
−3.2	.0007	.0007	.0006	.0006	.0006



6. The SAT scores of students in a college is normally distributed with a mean of 510 and a standard deviation of 30. The college waives tuition for the students with SAT scores in top 10%. What is the lowest SAT score a student can have and still get the tuition waiver?

$0.1 = P(X > a) = 1 - P(X < a) = 1 - P(z < b)$

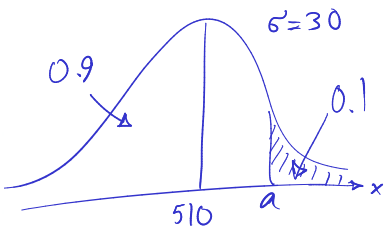
$\Rightarrow P(z < b) = 0.9$

$\Rightarrow z = 1.28$

$X = z \cdot \sigma + \mu$

$= (1.28)(30) + 510$

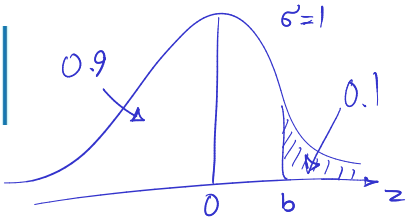
$= 548.4$



z	.00	...	.07	.08	.09
0.0	.5000	...	.5279	.5319	.5359
⋮	⋮		⋮	⋮	⋮
1.1	.8643	...	.8790	.8810	.8830
1.2	.8849	...	.8980	.8997	.9015
1.3	.9032	...	.9147	.9162	.9177

distance from 0.9: 0.0003 (smaller) 0.0015

$\rightarrow b = 1.28$



7. About 20% of U.S. drivers are uninsured. You randomly select 80 U.S. drivers and ask them whether they are uninsured. Find the probability that at most 50 of them are uninsured.

$P(X \leq 50) \approx P(X < 50.5)$

Binomial — Normal

$= P(z < 9.63)$

$= 1$

$np = 80 \cdot \frac{20}{100} = 16 \geq 5$

$nq = 80 \cdot \frac{80}{100} = 64 \geq 5$

$\rightarrow \mu = 80 \cdot \frac{20}{100} = 16$

$\sigma = \sqrt{80 \cdot \frac{20}{100} \cdot \frac{80}{100}} = \sqrt{12.8} \approx 3.58$

$z = \frac{50.5 - 16}{3.58} \approx 9.63 > 3.49$

Optional problems

8. How many distinguishable words are there using the letters of "H A L L O W E E N".

9. The heights (in feet) and the numbers of stories of nine buildings in Houston are listed. Use a scatter plot to display the data. Describe any patterns.

Height (in feet)	992	780	762	756	741	732	714	662	579
Number of stories	71	56	53	55	47	53	50	49	40