Stat 171 - Exam 2

SHOW ALL OF YOUR WORK

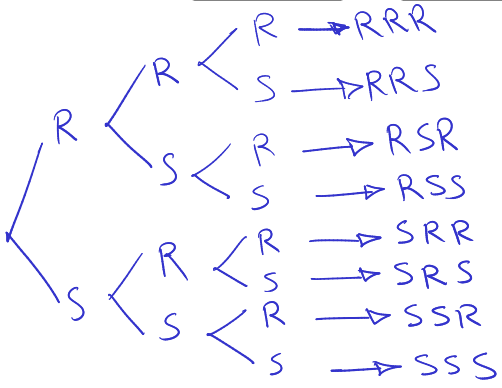
Name: Solution

You are planning a three-day trip to Seattle, WA, in October. Assume each day is going to be either **S**unny or **R**ainy.

1. Recording the weather condition of your three-day trip, give an example of an outcome.

RRS → first two days: rainy, last day: sunny

2. Draw a tree diagram and list the sample space for the weather during the three days of your trip.



$S = \{RRR, RRS, RSR, RSS, SRR, SRS, SSR, SSS\}$

3. Let E be the event that exactly two of the three days being sunny. What is the theoretical probability of E? Explain.

$E = \{RSS, SRS, SSR\} \rightarrow \#E = 3$, $\#S = 8$ $\rightarrow P(E) = \frac{\#E}{\#S} = \frac{3}{8}$

4. What is the complement of E? What is its probability?

E': the event that number of sunny days is not 2, i.e. no. of sunny days = 0 or 1 or 3.

$E' = \{RRR, RRS, RSR, SRR, SSS\}$ (0, 1, 1, 1, 3)

$P(E') = \frac{\#E'}{\#S} = \frac{5}{8}$, or $P(E') = 1 - P(E) = 1 - \frac{3}{8} = \frac{5}{8}$

5. Let A be the event that the first two days were sunny, and B be the event that the last two days were sunny.

   (a) Find $P(B \mid A)$. Are A and B independent? Why?

$A = \{SSS, SSR\}$

$B = \{SSS, RSS\}$

$P(B \text{ and } A) = P(SSS) = \frac{1}{8}$

$P(B|A) = \frac{P(B \text{ and } A)}{P(A)} = \frac{1/8}{2/8} = \frac{1}{2}$

$P(B) = \frac{2}{8} = \frac{1}{4} \neq \frac{1}{2} = P(B|A) \Rightarrow$ A and B are not independent.

6. The table shows the results of a survey that asked 1048 U.S. adults whether they support or oppose a special tax on junk food. A person is selected at random from the sample. Find the probability of each event.

	Support	oppose	Unsure	Total
Male	163	325	5	493
Female	396	300	22	555
Total	396	625	27	1048

(a) The person opposes the tax.

A

$$P(A) = \frac{625}{1048}$$

(b) The person is female.

B

$$P(B) = \frac{555}{1048}$$

(c) The person opposes the tax **and** is female.

$$P(A \text{ and } B) = \frac{300}{1048}$$

(d) The person opposes the tax **or** is female.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{625}{1048} + \frac{555}{1048} - \frac{300}{1048}$$

7. There are 10 finalists in a singing competition. The top 3 singers receive prizes.

(a) How many ways can the singers receive a prize?

no order → combination

$$_{10}C_3 = \frac{10!}{7!\,3!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}\; 3 \cdot 2 \cdot 1} = 120$$

(b) How many ways can the singers finish first through third?

order matters → permutation

$$_{10}P_3 = \frac{10!}{7!} = 10 \cdot 9 \cdot 8 = 720$$

Optional problems

8. A sample of people are asked how much they have donated to charity last year. Draw a frequency histogram labeled with class midpoints for the given data, using 5 classes.

$$215, 157, 122, 238, 354, 490, 483, 294, 383, 334, 259, 387, 100, 367, 406,$$
$$10, 424, 310, 384, 245, 226, 247, 367, 483, 94, 298, 57, 260, 201, 387.$$

Optional problems

9. The table shows the results of a survey that asked 1048 U.S. adults how much they donated to charity last year. Find the standard deviation of the data.

	0	**100**	**200**	**500**
Frequency	396	625	22	5