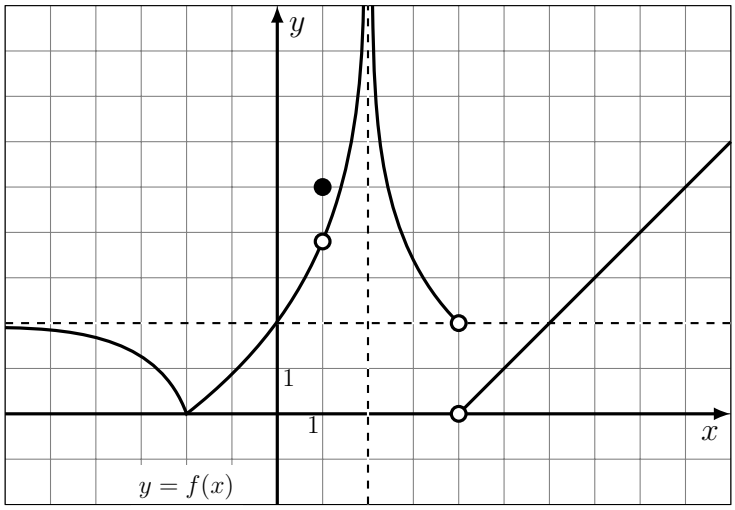
Math 133 - Exam 1

Name: _____ SHOW ALL OF YOUR WORK.

The graph of the function $y = f(x)$ is shown below.



1. Find the following limits for the function $f(x)$ shown above.

   (a) $\lim_{x\to-\infty} f(x)$

   (b) $\lim_{x\to\infty} f(x)$

   (c) $\lim_{x\to 2} f(x)$

   (d) $\lim_{x\to 0} \sqrt{f(x)}$

   (e) $\lim_{x\to 4^-} f(x)$

   (f) $\lim_{x\to 4^+} f(x)$

   (g) $\lim_{x\to 1} f(x)$

   (h) $\lim_{x\to 7} f(x)$

2. List all the discontinuity points of $f(x)$ shown above and list **ALL** reasons why f is discontinuous at that point.

Points (only x)	**Reasons** (Using the graph)	**Reasons** (Using the definition of continuity)

3. Evaluate the limit. Show all steps. Mention any theorems used. If the limit does not exist, explain why.

(a) $\lim_{x \to 2} \dfrac{3x^3 - 2x^2 + x}{x - 1}$

(b) $\lim_{x \to 1} \dfrac{x^3 - x}{x - 1}$

(c) $\lim_{x \to 5} \sqrt{5 - x}$

(d) $\lim_{x \to \frac{\pi}{2}} \cos^2(x)$

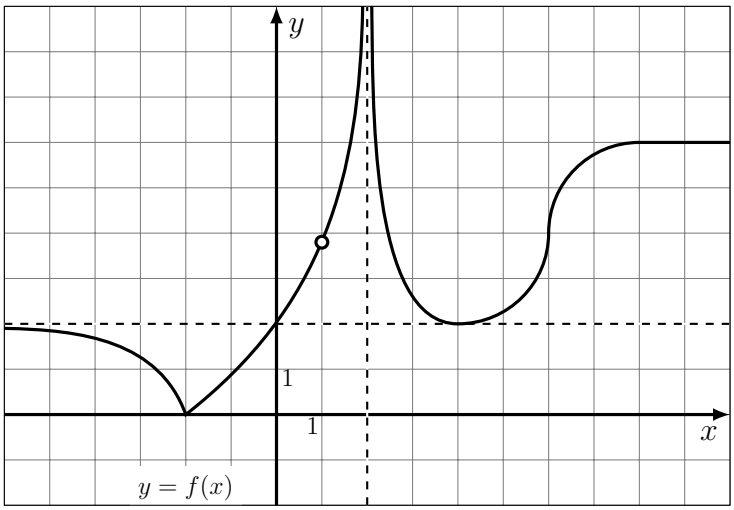
(e) $\lim_{x \to -\infty} \dfrac{(3x - 1)(2x^2 + 2)}{-x^3 + x^2 - x + 1}$

4. Find all the points where $f(x) = \ln(\frac{1}{x-1})$ is continuous. Explain your answer.

5. Show that the equation $x^4 + 5x^3 + 5x - 1 = 0$ has at least one real solution in the interval $(-1, 1)$.

6. Let $f(x)$ be a function such that $1 - x^2 \leq f(x) \leq \cos x$, for all x in the interval $[-\pi/2, \pi/2]$. Find $\lim_{x \to 0} f(x)$.

The graph of the function $y = f(x)$ is shown below.



7. Approximate the following derivatives for the function $f(x)$ shown above. If the derivative does not exist, explain why.

   (a) $f'(-4)$

   (b) $f'(-2)$

   (c) $f'(0)$

   (d) $f'(1)$

   (e) $f'(2)$

   (f) $f'(4)$

   (g) $f'(6)$

   (h) $f'(9)$

8. Let $f(x) = 1/x$.

   (a) Find $f'(x)$, the derivative function of f.

   (b) Find $f'(5)$.